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LETTER TO THE EDITOR

The effect of dynamics on damage spreading in the two-dimensional classical XY model

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Abstract. We study damage spreading in the classical two-dimensional XY model, using a dynamics and distance measure which preserve the rotational invariance of the Hamiltonian. We find only a high temperature random phase and a low temperature ordered phase, consistent with equilibrium results. Our results contrast to previous results of Golinelli and Derrida.

The notion of 'damage spreading', i.e. measuring the distance between two different initial spin configurations as they evolve in time according to the same dynamical rule subject to the same stochastic noise, has been used as a means of studying phase transitions in statistical systems [1–9]. Although it is a dynamical method, it has been argued that the transitions found with this approach often reflect a corresponding equilibrium transition [1, 3–5]. Hence the method has been applied in cases where equilibrium phase transitions are hard to detect in standard Monte Carlo simulations, such as the spin glass problem [2, 6] and commensurate–incommensurate transitions [7].

If the connection between damage spreading and equilibrium phase transitions is to be strengthened, it is important to understand the role of the particular dynamical rule used in the damage spreading calculation. Recently Mariz *et al* [8] have carried out a calculation of damage spreading in the two-dimensional ferromagnetic Ising model, comparing heat bath, Glauber, and various Monte Carlo dynamics. In this letter we consider the two-dimensional classical XY model, and show that the transitions in damage spreading depend crucially on the symmetry of the dynamics chosen.

In a recent paper, Golinelli and Derrida [9] applied the idea of damage spreading to study behaviour in the ordinary 2D classical XY model. They found surprising results, suggesting three separate phases. For a configuration of spins specified by their angles $\{\theta_i\}$, the Hamiltonian is

$$\mathscr{H} = -\sum_{\langle i,j\rangle} \cos(\theta_i - \theta_j) \tag{1}$$

where the sum is over nearest-neighbour sites of a square lattice of length L. Defining the distance at time t between two spin configurations $\{\theta_i(t)\}$ and $\{\tilde{\theta}_i(t)\}$ as

$$D(t) = \frac{1}{2L^2}\sum_i [1 - \cos(\theta_i(t) - \tilde{\theta}_i(t))] \tag{2}$$

Golinelli and Derrida find at large t that for $T > T_2 \sim 1.8$, the distance $D(t) = 0$, independent of the initial conditions; for $T_2 > T > T_1 \sim 1.2$, $D(t)$ approaches a non-zero

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constant, independent of the initial conditions; and for $T_1 > T$, $D(t)$ approaches a non-zero constant, which does depend on the initial conditions. Such a three-phase behaviour is similar to that seen in damage spreading calculations for the 3D Ising spin glass [2]. In the present case, T_1 lies close to the equilibrium Kosterlitz-Thouless transition temperature [10, 11], while Garel *et al* [12] have suggested that T_2 may be related to a disorder point.

We now show that this three-phase behaviour is a direct consequence of the particular dynamics chosen. When we modify the dynamics to preserve the rotational symmetry of the Hamiltonian (1), we recover only the two phases familiar from equilibrium simulations. This puts in question whether the T_2 found by Golinelli and Derrida does in fact correspond to any equilibrium phenomenon, or is rather a purely dynamical effect.

The dynamics of Golinelli and Derrida is summarized as follows: to update a configuration $\mathscr{C} = \{\theta_i\}$, a random site i is chosen, and a new configuration $\mathscr{C}'$ is formed by replacing the spin θ_i by a new one whose angle θ_i' is uniformly distributed on $[0, 2\pi]$. This new $\mathscr{C}'$ is then either accepted or rejected according to the standard Metropolis algorithm, i.e. accepted if $z \leq \exp[(\mathscr{H}(\mathscr{C}) - \mathscr{H}(\mathscr{C}'))/T]$ where z is a random number uniformly distributed on $[0, 1]$. The configuration $\tilde{\mathscr{C}} = \{\tilde{\theta}_i\}$ is then updated using the same Metropolis rule with the same site i, the same new spin θ_i', and the same random number z. This dynamics is similar to the heat bath dynamics [8], in that the new spin θ_i' is chosen independent of the old spin θ_i. One important feature of this dynamics is that it breaks the rotational invariance of the Hamiltonian (1), i.e. if the configuration $\{\theta_i(0)\}$ evolves into $\{\theta_i(t)\}$ then the configuration $\{\theta_i(0) + \phi_0\}$ does not in general evolve into $\{\theta_i(t) + \phi_0\}$. This is explicitly shown in Golinelli and Derrida's simulation: for $T > T_2$ two oppositely aligned initial configurations with $D(0) = 1$, evolve into configurations with $D(t) = 0$. If rotational symmetry were preserved, the distance between these two configurations would be a constant of the motion.

We consider now, instead, a dynamics for the XY model which does preserve rotational symmetry. Our updating scheme is just as described above, except now we choose the new spin to be $\theta_i' = \theta_i + \Delta\theta$ where $\Delta\theta$ is uniformly distributed on $[-\delta, \delta]$, with δ chosen as a function of temperature to give roughly a 50% acceptance. By defining the new spin in terms of a rotation of the old spin, rotational invariance is preserved. Furthermore, since the two configurations $\{\theta_i\}$ and $\{\theta_i + \phi_0\}$ now have identical equilibrium and dynamical behaviour, we redefine the distance function to measure zero distance between them; i.e. two configurations related by rotation of all spins by a constant angle, are now regarded as equivalent. We do this by first rotating all the spins of one configuration so that the total magnetizations of the two configurations are aligned, before applying the distance function (2). Equivalently, if

$$\alpha(t) = \tan^{-1}\left(\frac{\Sigma_i \sin(\theta_i(t))}{\Sigma_i \cos(\theta_i(t))}\right) \tag{3}$$

is the angle of the total magnetization of $\{\theta_i(t)\}$, then our new distance function is

$$\tilde{D}(t) = \frac{1}{2L^2}\sum_i [1 - \cos(\theta_i(t) - \tilde{\theta}_i(t) - \alpha(t) + \tilde{\alpha}(t))]. \tag{4}$$

Using our rotationally invariant dynamics and measure, we now repeat the damage spreading calculation. We consider three different initial conditions for the two initial configurations $\mathscr{C}(0)$ and $\tilde{\mathscr{C}}(0)$.

(a) $\mathscr{C}(0)$ is random. $\tilde{\mathscr{C}}(0)=\mathscr{C}(0)$ except for 10% of the spins which are randomly chosen and given random angles.

(b) Same as in (a) except 20% of the spins are different.

(c) $\mathscr{C}(0)$ and $\tilde{\mathscr{C}}(0)$ are chosen randomly and independently.

We study square lattices of length $L=8$, and 16. Our results are shown in figure 1, for $\bar{D}(t)$ against T, as $t=1500$ time steps, after equilibrium has been achieved (each time step represents an update of L^2 spins). We average over 100 different initial configurations, for each of the three conditions above. At high temperatures T, the distance $\bar{D}(t)=\frac{1}{2}$, indicating that the two configurations have become completely random with respect to each other. At low T, $\bar{D}(t)\to 0$, indicating that the two configurations have become identical. The transition between these two limits occurs in a temperature region which gets narrower as L increases. At all temperatures, there is no memory of the initial condition (a)-(c). We have also done simulations for $L=32$, with condition (a). Our results, at $t=2500$, averaged over 10 different initial configurations, are also shown in figure 1. They show the same trends as above. The increasing tail at very low T is due to the failure to reach equilibrium.

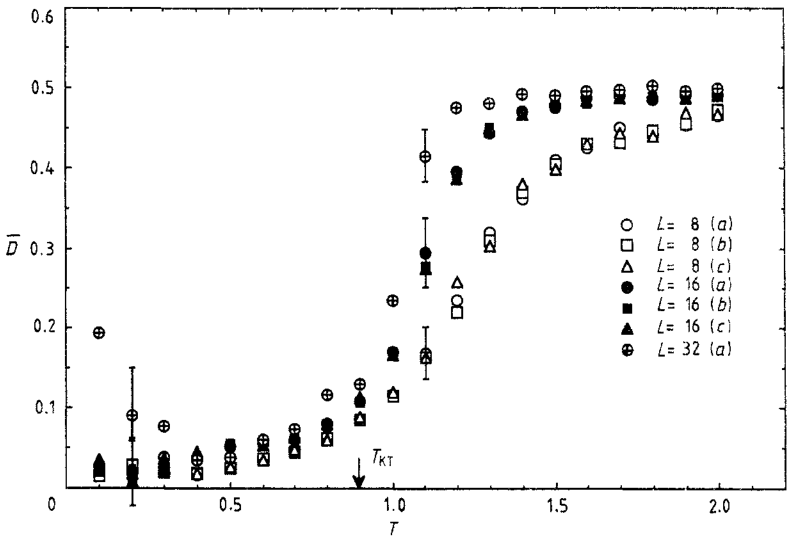

Figure 1. Damage spreading distance $\bar{D}(t)$ against temperature T, for various lattice lengths L and initial conditions (see text) (a), (b), (c) as shown. For $L=8$, 16 $\bar{D}$ is evaluated at $t=1500$; for $L=32$, at $t=2500$. No memory of initial conditions is apparent. The Kosterlitz-Thouless equilibrium transition temperature, $T_{KT}\simeq 0.89$, is indicated for comparison. Typical error bars are shown at $T=0.2$ and 1.1.

We thus see that once rotational invariance is restored in the dynamics, we find only two phases characterizing different damage spreading behaviour: a high T random phase, and a low T ordered phase. Neither phase has any memory of initial conditions. The transition occurs close to the equilibrium Kosterlitz-Thouless transition $T_{KT}\sim 0.89$, and we note that the quantity $1-2\bar{D}$ bears a striking resemblance to the finite-size behaviour of the equilibrium helicity modulus [13] (although we have no theoretical argument connecting the two). Our calculations suggest that damage spreading transitions may be more closely related to equilibrium transitions, when the dynamics chosen preserves the symmetry of the Hamiltonian.

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